ABSTRACTS OF ARTICLES DEPOSITED AT VINITI*

THEORY OF HEAT- AND MASS-EXCHANGE (THERMOGRAPHIC) METHODS OF RESEARCH

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UDC 541.11

The presently adopted means of analysis of thermograms and derivatograms use far from all the information on the properties of the specimen which these curves contain. The present report is devoted to the derivation and analysis of equations connecting a thermogram and derivatogram with the equilibrium and transfer characteristics of a disperse substance and the conditions of its drying.

The equations obtained in the report show that the form of the thermogram and derivatogram is determined by three groups of factors: the motive force of mass transfer, i.e., the difference in chemical potentials in the specimen and air, the resistance to the motion of liquid and vapor in the specimen, and finally, the conditions of external mass exchange in the surrounding medium. Thus, the individual properties of the object of study, reflected by the form and magnitude of the thermogram and derivatogram, depend on two characteristics of the specimen: the chemical potential of the bound liquid in the specimen (the equilibrium characteristic) and the mass conductivity of the specimen (the transfer characteristic). In this case the experimental conditions, as follows from the equations obtained, determine which properties of the specimen — equilibrium or transfer properties — most strongly affect the form of the thermogram and derivatogram.

The equations obtained make it possible to solve two basic problems of thermographic analysis. The first consists of finding the thermogram and derivatogram from the known properties of the specimen. This in turn makes it possible to make a qualitative or semiquantitative estimate of the properties of the specimen from the characteristic sections of the thermogram (straight, exponential, etc.). The second problem consists of the quantitative determination of the characteristics of the material from the known thermogram or derivatogram.

Typical cases of the application of the equations obtained for the analysis of thermograms and derivatograms of drying are considered.

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THERMOPHYSICAL PROPERTIES OF CARBON-FILAMENT MATERIALS IN THE RANGE OF 10-400°K

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UDC 536.48.083

The coefficients of thermal conductivity, thermal diffusivity, and heat capacity of carbon-filament materials along and across the filament — filler in the temperature range of 10-423°K are studied in the report.

The method and the experimental setup are described in sufficient detail in [1]; it should only be noted that the study of the thermal properties of the materials was performed by the quasisteady composite method. The maximum error was 7-8%. The thermophysical properties of a carbon-filament material (specific weight

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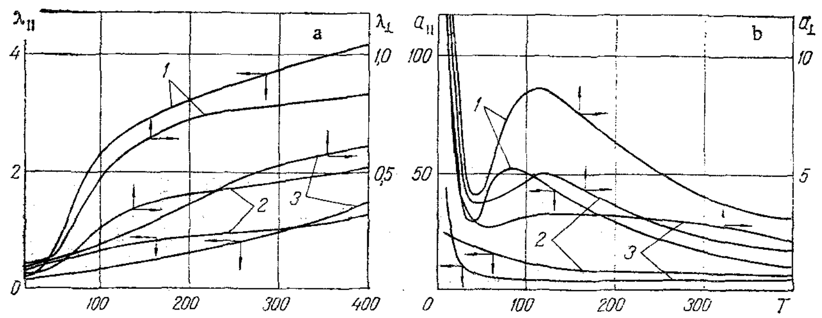

Fig. 1. Dependence of thermal conductivity of carbon-filament materials along $\lambda_{\parallel}$ and across $\lambda_{\perp}$ the filament (a) and thermal diffusivity of carbon-filament material along $a_{\parallel}$ and across $a_{\perp}$ the filament (b) on temperature: 1) after single cooling; 2) after fivefold thermal shock; 3) after 10-fold thermal shock. T, °K.

$1{,}44 \cdot 10^3$ kg/m^3, volumetric content of resin 31,8%, degree of solidification of resin 98.9%) were studied as a function of the temperature, number of cycles of temperature shocks (5 and 10), and the direction of the heat flux (along $\parallel$ and across $\perp$ the filler). The experimental data are presented in Fig. 1. As seen from the figure, the coefficient of thermal diffusivity a behaves in an anomalous way. The anomaly is expressed in the appearance of a maximum, more expressed for the specimen along the filament, in the region of hydrogen — nitrogen temperatures. The temperatures at which the curves $a_{\parallel}$(T) and $a_{\perp}$(T) have maxima coincide with the points of inflection of the curves $\lambda_{\parallel}$(T) and $\lambda_{\perp}$(T).

The temperature dependence of $a_{\parallel}$ and $a_{\perp}$ is explained by the peculiar temperature dependence of the phonon mean free path of the composite. One can assume that two processes take place in parallel in the material under study, from 40 to 80°K along the filament and from 40 to 120°K in the case of the transverse arrangement of the filament, under the effect of residual and thermal stresses caused by the anisotropy of the coefficient of thermal linear expansion of the carbon-filament material (negative along the filament and positive across it) and the considerable difference between the thermal coefficients of the filament and the binder: the formation of higher-order supermolecular formations from the microfibrils of the carbon filament and the transition of the binder film into a crystalline state with the formation of crystallites.

These new supermolecular structures and crystallites possess considerable anisotropy of properties along and across the filament and along and across the binding film, respectively.

Since the developing anisotropy of properties is a consequence of thermal stresses whose magnitude and direction depend on the temperature range, it also has a variable character.

Dependences of the coefficient of thermal conductivity λ of the carbon-filament structure on the temperature following cycles of temperature shocks (5 and 10) are presented in Fig. 1.

As seen from the figure, the values of the thermal conductivity decreased by almost five times following five cycles in comparison with the initial data. This is explained by the fact that cracking of the binding film in the radial direction with respect to the carbon filament and splitting of the fibrils of a carbon filament into microfibrils take place under the effect of the thermal stresses arising under the effect of the cycles of thermal shocks.

Following 10 cycles the dependences λ(T) acquire the form characteristic of the λ(T) curves of dry dispersed media, i.e., the carbon-filament material changes from a monolithic material into a bound dispersed system with open pores.

The coefficients of thermal diffusivity also change considerably under the effect of thermal shocks: the maximum of thermal diffusivity disappears and following 10 cycles the dependences $a_{\parallel}$(T) and $a_{\perp}$(T) also have the form characteristic of the coefficient of thermal conductivity of dispersed bound systems with open pores.

Experimental data on the dependence of the specific heat capacity on the temperature and cycles of thermal shocks are also presented in the report.

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HEAT EXCHANGE DURING BOILING OF SEAWATER IN A FILM FLOWING DOWN OVER A HORIZONTAL PIPE

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UDC 621.1.016.4

Heat exchange during the boiling of seawater is studied on an experimental installation containing an evaporator made of a vertical array of horizontally arranged pipes and a sprinkler [1]. The tests established that the dependence of the heat-transfer coefficient α_2 remains constant in the entire range of variation of the sprinkling densities Γ_V = 200-2000 kg/(m·h) for all values of the heat fluxes q = 10-130 kW/m^2, pressures p = 30-160 kPa, concentrations b = 3.5-16%, and pipe diameters d_0 = 16, 24, and 32 mm.

In this case with an increase in the heat flux one observes an increase in the average values of α_2 with retention of the form of the dependence $\alpha_2 = f(\Gamma_V)$. For example, α_2 = 6000 W/m^2·°C for q = 30 kW/m^2 and α_2 = 10,000 W/m^2·°C for q = 100 kW/m^2. With q and b constant the values of the heat-transfer coefficient obtained are higher with an increase in pressure. The intensification of heat exchange during boiling in a horizontal film depends to a greater degree on q, which determines the nature of the vapor formation. In the region of small q one observes an evaporation mode with very slight formation of vapor bubbles, while with an increase in q a transition to developed bubble boiling occurs. The transition zone is estimated by the quantity

$$q' = 95.8p^{-0.2}\left(1+\frac{b}{100}\right)^{2.3}. \tag{1}$$

Here the heat-transfer coefficient for seawater is

$$q < q',\ \alpha_2 \sim q^{0.25} \text{ and } q > q',\ \alpha_2 \sim q^{0.65},$$

which is explained by the strong foaming in the evaporation mode, the effect of which is suppressed during bubble boiling.

The presence of ordered motion of the liquid film leads to the premature washing off of the first bubbles which form, carried off by the film and destroyed beyond the limits of the heating surface.

The sizes of the vapor bubbles and their number are determined by both q and Γ_V. The generation of bubbles takes place at a certain film thickness, which depends on the pipe diameter and the sprinkling density, which comprises 400-600 kg/(m·h), corresponding to δ = 0.35-0.5 mm. The decrease in the heat-transfer coefficient with a decrease in pressure for q < q' is connected with an increase in the viscosity of the liquid, producing an increase in the thickness of the laminar wall boundary layer which creates the main thermal resistance. For the region of q > q' the vapor bubbles turbulize the boundary layer, and therefore a certain increase in the heat-transfer coefficient in comparison with the evaporation mode takes place with an increase in p, other conditions being equal. The sizes of the separation diameter of the vapor bubbles increase with a decrease in pressure [2]. An analysis of the data obtained shows that the stability of heat exchange depends for the most part on the minimum and maximum attainable sprinkling densities.

It is established that the quantity Γ_{min} which produces the required wetting of the heating surface without a break in the continuity of the film during its boiling is a function of both the thermal and the geometrical characteristics of the heating surface, and for the evaporation mode it can be represented by the equation

$$\Gamma_{\min} = 70.5q^{0.5}d_0^{0.3}\left(\frac{t}{d_0}\right)^{0.6}\left(1+\frac{b}{100}\right)^{1.7}$$

and for the bubble boiling by the equation

$$\Gamma_{\min} = 8.3q^{1.3}d_0^{0.6}\left(\frac{t}{d_0}\right)^{1.5}\left(1+\frac{b}{100}\right)^{1.1}, \tag{2}$$

whereas Γ_{max} is determined independently of q'

$$\Gamma_{\max} = 5.45\cdot 10^3 d_0^{0.92} \tag{3}$$

without the other parameters having an appreciable effect on its value.

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DETERMINATION OF TEMPERATURE OF A SPHERE AND THERMAL DIFFUSIVITY OF THE MEDIUM FROM TEMPERATURE MEASUREMENTS OUTSIDE THE SPHERE

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UDC 536.24.02

In a number of heat-conduction problems it becomes necessary to measure the temperature as a function of time at points lying outside a certain volume and to reconstruct the temperature at the boundary of this volume as a function of time.

Often the problem is still more complicated because the heat-conducting properties of the medium also turn out to be unknown. Problems of such a type arise, in particular, if the volume at whose boundary the temperature is reconstructed consists of a sphere formed at large depths with the help of an explosion.

In addition, the medium surrounding the cavity is often not uniform itself, and therefore the assumption that it is uniform is a certain approximation within the framework of which one must determine the optimum coefficient of thermal diffusivity of the homogenized medium such that one obtains the least discrepancy between the solution of the problem on the assumption of uniformity of the medium and the solution within the framework of a model with allowance for the nonuniformity of the real medium.

In the present report the criterion for choosing the optimum coefficient of thermal diffusivity a is the minimization with respect to a of the nonnegative functional

$$J(a) = \int_0^{\tau^*} [\max_{x=x_1,\ldots,x_n} T_S(\tau, x, a) - \min_{x=x_1,\ldots,x_n} T_S(\tau, x, a)]\, d\tau,$$

where $[0, \tau^*]$ is the time interval of interest to us; T_S is the temperature at the surface of the sphere; $x_1, \ldots, x_n$ are the distances from the sphere to the points where the temperature is measured.

For each fixed set of values of (x, a) the problem of the determination of T_S as a function of the time $\tau \in [0, \tau^*]$ comes down to the solution of an integral equation of the first kind, and consequently it belongs to the class of improper problems.

The regularized method of successive approximations is used in the report for the solution of the integral equation given above. On the basis of the proposed method an alogorithm was developed and a computer program was written for the solution of the problem stated above.

An analysis of the numerical results obtained above and their comparison with experimental data showed the efficiency and reliability of the proposed method.

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UNSTEADY HEAT EXCHANGE DURING THE MOTION OF POWER-LAW FLUIDS IN THE CASE OF VARIATION OF THE HEAT-FLUX DENSITY AT THE WALL OF A CHANNEL AND A PIPE

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UDC 536.252

The present report is devoted to a study of heat exchange in the steady laminar mode of motion of a power-law fluid in the thermal initial section of a flat channel and of a round cylindrical pipe in the case of variation in the heat-flux density at the wall.

In this case the problem comes down to the solution of the differential energy equation in dimensionless form

$$\frac{\partial (T-T_0)}{\partial \mathrm{Fo}} + A_\omega [1-(1-Y)^{\frac{n+1}{n}}] \frac{\partial (T-T_0)}{\partial Z} = \frac{\partial^2 (T-T_0)}{\partial Y^2} - \frac{\omega}{(1-Y)} \cdot \frac{\partial (T-T_0)}{\partial Y},$$

where $A_0 = (2n+1)/(n+1)$ for $\omega = 0$, $A_1 = (3n+1)/(n+1)$ for $\omega = 1$, T is the temperature of the moving medium, T_0 is the temperature at the entrance to the pipe, Fo is the Fourier parameter, Y and Z are dimensionless coordinates, and n is the nonlinearlity parameter.

The problem is solved by the boundary-layer method in conjunction with the method of characteristic curves, for which the temperature distribution over the thickness δ of the boundary layer was assigned in the form of a polynomial whose coefficients were determined from the boundary conditions $\partial(T - T_0)/\partial Y = -q_w h/\lambda$ for Fo > 0 and Y = 0 while $T - T_0 = 0$ and $\partial(T - T_0)/\partial Y = 0$ for Fo > 0 and $Y = \delta$, where q_w is the heat flux density at the wall, 2h is the distance between the plates of the channel, and λ is the coefficient of thermal conductivity.

The calculations showed that the time needed for the establishment of the temperature field is less for pseudoplastic media than for viscous and dilatant media. In this case a change in the nonlinearity parameter n has an important effect on the time of establishment of the process for pseudoplastic media whereas this change is insignificant for dilatant media. The foregoing is obviously explained mainly by the radical reorganization of the temperature field, which is connected with a strong increase in the velocity gradients, for dilatant media in comparison with viscous and pseudoplastic media.

The time of establishment of the temperature field for different n strongly depends on the length of the initial thermal section.

This dependence increases with an increase in Z, other conditions being equal.

An example is given of calculations of the time of establishment of a steady temperature field and of the wall temperature at a certain distance from the start of the heated section at any time.

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SOME DATA ON THE HYDRAULIC RESISTANCE OF A PIPE TO THE MOTION OF A HIGHLY VISCOUS LIQUID WITH THE INJECTION OF A SOLVENT THROUGH THE POROUS WALL

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In the transportation of highly viscous liquids the problem arises of reducing the friction near the channel surface. Among the numerous means of affecting the liquid boundary layer [1-10] injections of a low-viscosity liquid through the porous channel wall find application [5, 6]. The study of the flow in pipes with mass exchange through the wall is of wider interest in connection with problems of swamp drainage, subterranean irrigation, the condensation or evaporation of liquids from the inner surfaces of heat exchangers, etc. Data of an experimental study of the hydraulic resistance of a round pipe to the motion of a viscous liquid with the injection of a solvent through the porous channel wall are presented below.

The experimental setup consists of a system of parallel pipelines with the same inner diameters and lengths. One of them was made of solid copper tubing with an inner diameter of 4 mm and a length of 1 m. The second consisted of a coaxial system of pipes. The inner pipe with an inner diameter of 4 mm and a wall thickness of 1 mm was made of porous Nickonel (penetration factor $K = 10^{-10}$ m). The outer pipe (of solid metal) left an annular gap of 2 mm between the pipes. A solvent (kerosene) was injected into the space between pipes. A viscous liquid (oil) moved through the inner pipe. The operating parameters were varied in the following ranges: $60 \leq Re \leq 523$, $0 \leq Re_w \leq 18$. The pressure at the inlets and outlets of the pipelines, the temperature, the volumetric flow rate of the transported liquid, and the total flow rate per second of solvent through the pipe surface were measured in the experiments.

The experimental data on the liquid flow through the unary pipe obey the Hagen — Poiseuille law.

The experimental results on the flow of liquid through a porous channel with the injection of a solvent show that the flow rate can be increased up to 40% through the injection of no more than 0.5% of solvent with respect to the main flow.

On the basis of the data obtained it is found that the coefficient of hydraulic resistance is reduced about twofold with the injection of a solvent through the porous channel wall.

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HYDRAULIC RESISTANCE OF GRANULAR BEDS OF CRUSHED BAUXITE CAKES

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The contradictory data of many reports on the hydrodynamics of granular media are caused by the absence of a physical basis for the analogy of flows of fluids through granular media at equal Reynolds numbers. The conclusions of these reports are used without critical analysis by many engineering offices in applied calculations, on the basis of which design orders are issued for the development of instrumental — technological systems.

An experimental study of the hydrodynamics of granular media formed by crushed bauxite cakes made it possible to establish that for this particular case $\lambda Re = 450$ up to the maximum values of $Re \simeq 10.5$ reached in the present work.

The authors developed a method of determining the particle shape coefficient both for porous lump materials and for different kinds of column packings and natural soils.

In the determination of the bulk (apparent specific) weights of porous materials with particle sizes of less than 10 mm the method of hydrostatic suspension of the particles with their preliminary saturation by inert liquids (not entering into a chemical reaction with them) following saturation for two minutes makes it possible to drain the liquid from the test samples. In our case this turned out to be unsuitable. When kerosene was used as the inert liquid a thick film was left on the rough surface of the cake grains, while in the case of the use of benzene it evaporated rapidly. In either case the convergence in seven parallel tests was poor. This difficulty could be overcome by removing the excess liquid with filter paper. Another novelty introduced into the method was the use of a pycnometer with two ground-glass joints: base—throat and throat—stopper. With the throat removed the smallest diameter of the opening in the base of the pycnometer was 12 mm, and thus it was easy to introduce cake grains (the maximum fraction studied was −10 + 8 mm).

The number of particles in the sample was counted down to the fraction of −0.8 + 0.63 mm inclusively. Knowing the volume of the sample of a given narrow fraction of cake and the number of particles in it, the average volume per particle was calculated, and taking it as a sphere the nominal particle diameter for the given fraction was found and from it the specific surface S_1.

The shape factor Φ, which allows for the dependence of the equivalent diameter of a particle on its shape, was determined from the equation

$$\Phi = \frac{S_2}{S_1}, \tag{1}$$

where S_2 is the specific surface of particles of the bed from the average particle diameter of the fraction on the basis of a sieve analysis.

The particle shape coefficient was determined from the equation

$$\varphi_{\Phi} = \frac{1}{\Phi^2}. \tag{2}$$

Values of φ both smaller and larger than unity were obtained for bauxite cakes, which can pertain to aluminosilicates. The authors have not encountered values of $\varphi_{\Phi} < 1$ in the literature.

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STATIONARY FRONT IN THE MOVEMENT OF A MIXTURE THROUGH A POROUS MEDIUM WITH ALLOWANCE FOR HEAT RELEASE

L. K. Tsabek

UDC 532.546

Numerical and analytical solutions of the equations of isothermal and nonisothermal sorption dynamics for isotherms and thermal sorption functions of arbitrary form are analyzed for model A (the pore size of the porous grain is far larger than the sizes of the sorbate molecules) and model B (the pore size of the porous grain is comparable with the sizes of the sorbate molecules). It is shown that with infinitely large values of the coefficients of heat and mass exchange the system of equations of motion of a mixture through a porous medium admits of the existence of only two types of invariant solutions: solutions of the traveling-wave type and self-similar solutions of the spreading-wave type. For the existence and uniqueness of self-similar solutions of the spreading-wave type in the isothermal and nonisothermal cases, respectively, in the presence of a thermal sorption equation f(c, T) of arbitrary form it is necessary and sufficient to satisfy the following conditions:

$$\begin{gathered} d^2f/dc^2 > 0,\ (\lambda_i - a)/Q + (\partial\lambda_i/\partial T)/(\partial\lambda_i/\partial c) > 0, \\ \lambda_{1,2} = B \pm (B^2 - aH)^{1/2},\ B = (a + H + QF)/2,\ H = (\partial f/\partial c)^{-1},\ F = -H\partial f/\partial T. \end{gathered} \tag{1}$$

The second condition (1) is a limitation on the thermal function of physical sorption. If the conditions (1)-(3) are not satisfied for any thermal function found from thermodynamic considerations then such a thermal function does not have physical meaning.

The sections of isotherms or thermal functions which admit of the existence of solutions of the traveling-wave type will be called convex sections. If $\lambda(c_i) \neq w_i \neq \lambda(c_{i+1})$ (w_i is the velocity of the traveling wave), then the necessary and sufficient conditions for convexity of a section $c_i \leq c \leq c_{i+1}$ of the thermal equation have the form

$$w_i = \min_p w_p, \ \lambda(c_i) < w_i < \lambda(c_{i+1}). \tag{2}$$

If

$$w_i = \lambda(c_i) = \lambda(c_{i+1}), \ H^{(2)}(c_i) = H^{(3)}(c_i) = \cdots = H^{(s-1)}(c_i) = 0,$$
$$H^{(2)}(c_{i+1}) = H^{(3)}(c_{i+1}) = \cdots = H^{(p-1)}(c_{i+1}) = 0,$$

then the conditions of convexity for a thermal equation of arbitrary form have the form

$$\begin{gathered}(c - c_i) H^{(s)}(c_i) < 0, \ (c - c_{i+1}) H^{(p)}(c_{i+1}) > 0 \ (s, p \text{ even}),\\ w_i = \min_\rho w_p, \ H^{(s)}(c_i) < 0, \ H^{(p)}(c_{i+1}) > 0 \ (s, p \text{ odd}),\\ H^{(p)}(c) = \sum_{m=0}^{p} C_p^m A_*^m \partial f^{(p)}(c, T)/(\partial c^{(p-m)} \partial T^m), \ A_* = Q/(w_i - a).\end{gathered} \tag{3}$$

In the isothermal case under the conditions (3) we have $A_*^m|_{m>0} = 0$. For the equations of isothermal and nonisothermal desorption dynamics the signs must be reversed in the inequalities (1)-(3).

Analytical expressions are found for invariant solutions of the traveling-wave and spreading-wave types of isothermal and nonisothermal sorption dynamics for isotherms and thermal equations of arbitrary form. The equations of sorption dynamics were integrated by the difference system using a computer for various isotherms of arbitrary form and different thermal equations of arbitrary form. It is shown that in the nonisothermal case for concave and linear thermal equations the solutions are represented in the form of invariant solutions of traveling waves and spreading waves simultaneously.

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NONADIABATIC DYNAMICS OF PHYSICAL SORPTION

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Numerical and analytical solutions of nonadiabatic sorption and desorption dynamics in the presence of thermal sorption functions of arbitrary form are analyzed for models A and B of one-component and multicomponent sorbing mixtures. It is shown that the system of equations of nonadiabatic sorption dynamics strictly admits of the existence of only the mode of traveling waves. The conditions of convexity or the conditions of existence of invariant solutions of the traveling-wave type and the conditions of concavity or the conditions of existence of self-similar invariant solutions of the spreading-wave type for thermal functions of arbitrary form are the same in the nonadiabatic and isothermal cases. The conditions of convexity for multicomponent nonadiabatic sorption dynamics have the form

$$\begin{gathered}\mu_i(c^{(p)}) > 1/w_i^{(p)} > \mu_i(c^{(p+1)}), \ c^{(p)} = c\,(y \to \infty),\\ \mu_{i-1}(c^{(p+1)}) > 1/w_i^{(p)} > \mu_{i+1}(c^{(p)}), \quad c^{(p+1)} = c\,(y \to -\infty),\\ w_i^{(p)} = \min_s w_i^{(s)}, \quad y = z - w_i^{(p)} t,\end{gathered} \tag{1}$$

where μ_i are the eigenvalues of the matrix $A_{mk} = \partial f_m(c, 0)/\partial c_k$. If

$$\begin{gathered}\mu_i(c^{(p)}) = 1/w_i^{(p)}, \quad \mu_i(c^{(p+1)}) = 1/w_i^{(p)},\\ G_m^{(2)}(c^{(p)}) = G_m^{(3)}(c^{(p)}) = \cdots = G_m^{(s-1)}(c^{(p)}) = 0,\\ G_m^{(2)}(c^{(p+1)}) = G_m^{(s)}(c^{(p+1)}) = \cdots = G_m^{(g-1)}(c^{(p+1)}) = 0,\\ G_m^{(s)}(c) = \sum_{m_p=1}^{n} \left(\prod_{p=1}^{s} R_{m_p}(c)\right) \partial^{(s)} f_m(c, 0) \Big/ \left(\prod_{p=1}^{s} \partial c_{m_p}\right),\\ R_m(c) = r_m^{(i)}(c)/r_1^{(i)}(c),\end{gathered} \tag{2}$$

where $r_m^{(i)}(c)$ is a right-handed eigenvector of the matrix A_{mk} corresponding to the eigenvalue μ_i, then the conditions of convexity have the form

$$\begin{gathered}(c_1-c_1^{(p)})\,G_m^{(s)}(c^{(p)})>0,\quad (c_1-c_1^{(p)})\,G_m^{(g)}(c^{(p+1)})<0\ \ (s,\ g\text{— even}),\\ G_m^{(s)}(c^{(p)})>0,\quad G_m^{(g)}(c^{(p+1)})<0\quad (s,\ g\text{— odd}),\\ m=1,\ 2,\ \ldots,\ n.\end{gathered}\tag{3}$$

For isothermal and nonadiabatic desorption dynamics the signs must be reversed in the inequalities (1) and (3). Analytical expressions are presented for calculating invariant solutions of the traveling-wave type and of the type of quasi-self-similar solutions of spreading waves for thermal equations of arbitrary form. The equations of nonadiabatic sorption and desorption dynamics for different coefficients of heat exchange with the external medium and the equations of nonadiabatic three-component sorption dynamics were integrated on a computer by a difference system. An analysis of the numerical solutions showed that the dependence of the temperature on the concentration in a traveling-wave front can be approximately described by a quadratic function. Solutions are found for the steady-state equations of motion of a sorbed multicomponent mixture through a moving porous medium in the adiabatic and nonadiabatic cases. It is shown that using the conditions (2) and (3) one can find the limiting exit concentrations at the exit from a moving porous medium.

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APPLICATION OF PERTURBATION THEORY TO THE CALCULATION OF THE DIFFUSION POTENTIAL AT AN ION-EXCHANGE MEMBRANE

V. G. Veresov

UDC 541.183.12

The complete system of equations describing the movement of ions in an ion-exchange membrane with allowance for the self-consistent field and on the assumption of ideal behavior of the ions in the membrane phase has the form [1]

$$\begin{gathered}j_i^+=u_iRT\frac{dc_i^+}{dx}+u_ic_i^+F\frac{d\varphi}{dx},\\ j_k^-=v_kRT\frac{dc_k^-}{dx}-v_kc_k^-F\frac{d\varphi}{dx},\\ \frac{d^2\varphi}{dx^2}=-\frac{4\pi F}{\varepsilon_m}\left(\sum_{j=1}^{m}c_i^+-\sum_{k=1}^{l}c_k^-+\omega\bar{X}\right),\\ \sum_i j_i^+=\sum_k j_k^-.\end{gathered}\tag{1}$$

One is able to transform (1) into an ordinary third-order differential equation which depends only on the potential:

$$(A^*-B^*)=-\alpha\frac{d^3\Psi}{d\xi^3}+\{(c_0^{+*}+c_0^{-*})+(A^*+B^*)\xi+\omega\bar{X}^*(\Psi-\Psi_0)\}\frac{d\Psi}{d\xi}+\frac{\alpha}{2}\left[\left(\frac{d\Psi}{d\xi}\right)^2-\left(\frac{d\Psi}{d\xi}\right)_0^2\right].\tag{2}$$

With small α ($\alpha=\lambda_D/d$), where λ_D is the Debye length and d is the thickness of the membrane, one is able to obtain an asymptotic solution of (2) using the methods of perturbation theory. The nontriviality consists in the singular character of the problem, which leads to the appearance of regions of the boundary-layer type near the membrane boundary. Confining oneself to the first two terms, one is able to write a uniformly valid expression for the potential of the inner membrane:

$$[\Psi-\Psi_0]_{u.v}=\frac{A-B}{A+B}\ln\frac{\dfrac{A-B}{A+B}+\dfrac{c_0^++c_0^-}{\omega\bar{X}}+\dfrac{(A+B)X}{RT\omega\bar{X}}+\Psi_{[0]}-\Psi_0}{\dfrac{A-B}{A+B}+\dfrac{c_0^++c_0^-}{\omega\bar{X}}}-$$

$$- \sqrt{\frac{RT\varepsilon_m}{4\pi F^2(c_0^+ + c_0^-)}} \left[\frac{FE_0}{RT} - \frac{A-B}{RT\,(c_0^+ + c_0^-)} \right] \times$$

$$\exp\left\{ \frac{A-B}{RT\omega\bar{X}} \int\limits_0^x \frac{dx}{\left\{ \frac{c_0^+ + c_0^-}{\omega\bar{X}} + \frac{(A+B)\,d}{RT\omega\bar{X}} + (\Psi_{[0]} - \Psi_0) \right\}^2} \right\} +$$

$$+ \sqrt{\frac{RT\varepsilon_m}{4\pi F^2(c_0^+ + c_0^-)}} \left[\frac{FE_0}{RT} \quad \frac{A-B}{RT\,(c_0^+ + c_0^-)} \right] \exp\left[- \sqrt{\frac{4\pi F^2(c_0^+ + c_0^-)}{RT\varepsilon_m}} \right].$$

Here $\Psi_{[0]}$ is the solution of the transcendental Teorell equation [2],

$$\Psi_{[0]} - \Psi_0 = \frac{RT}{F} \frac{A-B}{A+B} \ln \frac{\frac{A-B}{A+B} + \frac{c_0^+ + c_0^-}{\omega\bar{X}} + \frac{(A+B)\,x}{RT\omega\bar{X}} + \Psi_{[0]} - \Psi_0}{\frac{A-B}{A+B} + \frac{c_0^+ + c_0^-}{\omega\bar{X}}}.$$

NOTATION

$\overset{+}{j}_i$, $\bar{j}_k$, fluxes; u_i, v_k, mobilities; φ, potential; F, Faraday number; ε_m, dielectric constant; c_i, c_k, concentrations; $\lambda_D \equiv \sqrt{RT\varepsilon_m/8\pi F^2 c_1}$;

$$c_1 \equiv \gamma^+ \frac{[c_\infty + c_{-\infty}]}{2}; \quad c_\infty = \sum_i c_{i\infty}^+ = \sum_k c_{k\infty}^-; \quad c_{-\infty} \equiv \sum_i c_{i\infty}^+ = \sum c_{k\infty}^-;$$

$$\Psi \equiv \frac{F\varphi}{RT}; \quad \overset{*}{u}_i \equiv \frac{u_i}{u_1}; \quad \overset{*}{v}_k = \frac{v_k}{u_1}; \quad \xi \equiv \frac{x}{d};$$

X, concentration of immobilized ions in membrane; γ^+, γ^-, coefficients of distribution.

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NUMERICAL INTEGRATION OF THE EQUATION OF OSCILLATIONS OF A VISCOELASTIC MEDIUM

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The given problem came up in the study of processes of vibrational treatment of disperse systems in closed vessels [1-4].

The Stokes equation of isothermal motion of an incompressible, viscous, Newtonian liquid in projection onto the x axis, with allowance for the fact that $E(\partial^2 e/\partial x^2)$ is introduced in place of $\partial p/\partial x$ [5], where E is the elastic modulus of an elementary volume of medium and e is the displacement of an element of medium, takes the form

$$\frac{\partial v}{\partial t} = c^2 \frac{\partial^2 e}{\partial x^2} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right), \tag{1}$$

where $v = \partial e/\partial t$ is the velocity of movement of an elementary volume of medium; $c \approx \sqrt{E/\rho}$ is the velocity of sound in the medium; ρ is the density; ν is the coefficient of kinematic viscosity.

For the numerical integration of the equation obtained we represent it in the difference form, replacing the first and second derivatives by finite differences in dimensionless quantities, for which we designate

$$e = Au,\ v = A\omega w,\ \overline{\delta t} = \frac{2\pi}{K_t},\ \delta t = \frac{\overline{\delta t}}{\omega}, \tag{2}$$

where A is the amplitude of the oscillations; u is the dimensionless displacement of an elementary volume; ω is the dimensionless velocity; $\overline{\delta t}$ is the dimensionless step in time; δt is the integration step in time; K_t is the number of time steps in an oscillation period.

With allowance for (2) Eq. (1) takes the form

$$w_0^{+\delta t} = w_0 + \frac{\nu\overline{\delta t}}{\delta x^2\omega}\Delta_x^2 w + \frac{\nu\overline{\delta t}}{\delta y^2\omega}\Delta_y^2 w + \frac{c^2\overline{\delta t}}{\delta x^2\omega^2}\Delta_x^2 u. \tag{3}$$

The conditions of stability of the solution of an equation of the hyperbolic type (3) are represented in the form $c \le \delta x/\delta t$ [6], which contradicts the physical essense of the process of propagation of oscillations in a viscoelastic medium. We have therefore assumed that $\delta x = \delta y = c\delta t$.

In the numerical integration of (3) we took into account the limitations on the strength of the perturbing action on an element of a viscoelastic medium and on the movement of an elementary volume of medium. We also used the approximation of the second derivatives in (3) by difference equations, in which the approximating function is represented in the form of a total quadric obtained by the method of least squares, as well as a model of relaxation smoothing in a viscoelastic medium.

As a result of the studies we obtained a similarity criterion in the form $\overline{K} = \eta\omega/E$ making it possible to model the process of propagation of oscillations in a viscoelastic medium in the presence of nonlinear boundary conditions (η is the coefficient of dynamic viscosity).

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PROBLEMS OF HEAT CONDUCTION FOR A ROD WITH TIME-DEPENDENT COEFFICIENTS OF HEAT EXCHANGE AND THEIR APPLICATIONS

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UDC 536.2

We examine the problem of determining the temperature field with variable coefficients of heat exchange through the ends and lateral surfaces of a semibounded rod:

$$u_t = \omega u_{xx} - \nu(t)\,[u - \delta(t)] + \frac{\omega}{\lambda}\psi;\ t > 0;\ x > 0, \tag{1}$$

$$-\lambda u_x|_{x=0} = \alpha\,[t,\ u(t)]\,[\delta(t) - u(t)],\ u|_{t=0} = \mu(x), \tag{2}$$

where λ and ω are the coefficients of thermal conductivity and thermal diffusivity; α and ν are the variable coefficients of heat exchange; ψ is the density distribution of heat sources; $u(t) = u(0, t)$.

Designating the unknown heat flux (2) as $q(t) = -\lambda u_x|_{x=0}$, using the Green function $G(x, \xi, t)$ of the second boundary problem of heat conduction we write the integral representation of the problem:

$$u(x,\ t)=F(x,\ t)+\frac{\omega}{\lambda}\int\limits_0^t\frac{q(\tau)\,h(t)}{h(\tau)}G(x,\ 0,\ t-\tau)\,d\tau. \tag{3}$$

Here F(x, t) is a known function; $h(t) = \exp\left[-\int\limits_0^t \nu(\tau)d\tau\right]$.

Substituting into (3) the heat flux through the right side of Eq. (2) and taking x = 0, we obtain a nonlinear integral equation with a weak singularity relative to the temperature:

$$u(t)=F(t)+\beta\int\limits_0^t\frac{\alpha[\tau,\ u(\tau)]\,h(t)}{h(\tau)\sqrt{t-\tau}}[\delta(\tau)-u(\tau)]\,d\tau, \tag{4}$$

where F(t) = F(0, t); $\beta = (1/\lambda)\sqrt{\pi/\omega}$.

For the solution of Eq. (4) we propose an approximate recurrent method which is convenient for making numerical calculations. The unknown function u(t) and all the known functions are represented in the form of step functions:

$$u(t)=\sum_{i=0}^{n-1}u(t_i)\,[E(t-t_i)-E(t-t_{i+1})], \tag{5}$$

where E(t) is a unit function. Substituting (5) into Eq. (4) and taking $t = t_n$, we obtain

$$u(t_n)=\frac{\omega}{\lambda}\sum_{i=0}^{n-1}\frac{h(t_n)}{h(t_i)}\alpha[t_i,\ u(t_i)]\,[\delta(t_i)-u(t_i)]\,\gamma(t_n,\ t_i). \tag{6}$$

Here

$$\gamma(t_n,\ t_i)=\int\limits_{t_i}^{t_{i+1}}\frac{d\tau}{\sqrt{t_n-\tau}}\simeq\frac{1}{\sqrt{t_n}}(t_{i+1}-t_i).$$

Setting n = 0, ..., n successively in (6), we obtain a system of recurrent equations of triangular form from which the values of $u(t_i)$ are determined successively, starting with the first $u(t_0) = F(t_0)$, after which the heat flux is found from Eq. (2) and then the thermal field from (3).

The boundary problem with variable coefficients of heat exchange at the boundaries of a finite rod is analyzed similarly. With certain assumptions the solutions obtained were used to find the thermal conditions of gas turbine blades operating in transitional regimes.

The results of the analytical calculations were compared with data obtained by modeling.

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SOLUTION OF MIXED PROBLEMS OF STEADY HEAT CONDUCTION FOR COMPOSITE CIRCULAR REGIONS IN THE PRESENCE OF NONIDEAL THERMAL CONTACT

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UDC 536.24

Many problems of thermophysics which are of practical interest are connected with the study of heat-conduction processes in regions possessing circular symmetry. The determination of the steady heat field of a circular cylinder is a well-studied problem. There are a number of problems, however, which have been little studied before, such as problems connected with the determination of heat fields in nonideally conjugate regions with variable boundary conditions.

In the present report we examine the problem of determining the steady temperature field in a piecewise-orthotropic thermally insulating cylinder in the presence of nonideal nonuniform thermal contact at the contact surfaces:

$$\frac{\lambda_\rho^{(i)}}{\rho}\ \frac{\partial}{\partial\rho}\left(\rho\frac{\partial u^{(i)}}{\partial\rho}\right)+\frac{\lambda_\varphi^{(i)}}{\rho^2}\ \frac{\partial^2u^{(i)}}{\partial\varphi^2}=-P^{(i)}(\rho,\ \varphi),\quad \rho\in(\rho_{i-1},\ \rho_i),\ i=\overline{1,\ N};\ \varphi\in(0,\ 2\pi)$$

with the boundary conditions

$$n_{11}(\varphi)\, u^{(1)}(\rho_0,\ \varphi) + n_{12}(\varphi) \frac{\partial u^{(1)}}{\partial \rho}(\rho_0,\ \varphi) = f_1(\varphi),$$

$$n_{21}(\varphi)\, u^{(N)}(\rho_N,\ \varphi) + n_{22}(\varphi) \frac{\partial u^{(N)}}{\partial \rho}(\rho_N,\ \varphi) = f_2(\varphi)$$

and the conjugation conditions $(\rho = \rho_i)$

$$u^{(i+1)} - u^{(i)} = \lambda_\rho^{(i)} R_c^{(i)} \frac{\partial u^{(i)}}{\partial \rho} + \Psi^{(i)}(\varphi),$$

$$\lambda_\rho^{(i+1)} \frac{\partial u^{(i+1)}}{\partial \rho} - \lambda_\rho^{(i)} \frac{\partial u^{(i)}}{\partial \rho} = X^{(i)}(\varphi),$$

$$i = \overline{1,\ N-1},$$

where $\lambda_\rho^{(i)}$ and $\lambda_\varphi^{(i)}$ are assigned parameters of the problem; $P^{(i)}$, $n_{p,q}$, f_p, $\Psi^{(i)}$, and $X^{(i)}$ are known piecewise-smooth functions; $R_c^{(i)}$ is the contact resistance of the i-th layer.

The initial problem is solved using a finite integral transformation with respect to the variable φ. In the image space the problem is reduced to an infinite system of linear algebraic equations which are solved by the reduction method. The convergence of the approximate solution is demonstrated.

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